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TRANSIENT HEATING OR COOLING OF A PLATE BY COMBINED CONVECTION AND RADIATION

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Abstract—Transient heat conduction in a plate subjected to heating and/or cooling by combined convection and radiation has been studied. The plate is assumed to be homogeneous, isotropic, and opaque to thermal radiation. The initial temperature distribution and the ambient environment as well as fluid temperatures are considered to be constant. In the analysis the transient heat-conduction equation and the boundary conditions are transformed to a nonlinear Volterra integral equation of the second kind for the surface temperature. This equation is solved numerically on a digital computer by the method of successive approximations. Precise results accurate to at least four significant figures have been obtained for a range of parameters of physical interest and are presented graphically as well as compared with approximate solutions and results available in the literature. Asymptotic solutions for small and large times are also given and their range of validity discussed.

NOMENCLATURE

$\mathscr{F}_{se}$, Hottel's radiation exchange factor between solid and environment;
q, surface heat flux;
g, dimensionless surface heat flux;
h, heat-transfer coefficient;
k, thermal conductivity;
L, half thickness of plate;
N_{Bi}, Biot number, hL/k;
N_{rc}, radiation number for cooling, $\mathscr{F}_{se}\sigma T_i^3 L/k$;
N_{rh}, radiation number for heating, $\mathscr{F}_{se}\sigma T_a^3 L/k$;
T, absolute temperature;
T_f, ambient fluid temperature;
T_e, environment temperature;
T_a, adiabatic surface temperature;
T_i, initial plate temperature;
T_s, surface temperature;
t, dimensionless time (Fourier number), $\alpha t_1/L^2$;
t_1, time;
u, dimensionless temperature defined in Table 1;
u_i, dimensionless initial temperature;
u_s, dimensionless surface temperature;
$w_i^{(2n)}$, Gaussian weights of order $2n$;
x, dimensionless space coordinate, x_1/L;
x_1, space coordinate;
x_i, Gaussian abscissa;
$y(t)$, dimensionless surface temperature.

Greek symbols

α, thermal diffusivity;
θ_c, T_a/T_i;
θ_h, T_i/T_a;
σ, Stefan–Boltzmann constant.

INTRODUCTION

THE PROBLEM of transient heat conduction in a solid becomes nonlinear when the thermophysical properties are dependent on temperature, or when either the heat source or the surface heat flux is a nonlinear function of temperature. Unfortunately, in the world of physical reality these nonlinear problems are commonplace and as with most nonlinear problems, they do not lend themselves to closed formed solutions.

This study is concerned with a nonlinear transient heat-conduction problem resulting from a nonlinear surface flux. In general an opaque solid is subjected to combined convective and radiative heat fluxes at the surface. When one mode of energy transfer at the surface predominates the other, two limiting cases arise: pure thermal radiation and pure convection. The radiation boundary condition is obviously always nonlinear and the convection boundary condition is nonlinear except for the very important case of forced convection with the heat-transfer coefficient independent of surface temperature. In many engineering problems dealing with transient heat conduction the radiative and convective surface fluxes are of the same order of magnitude. Thus, the transient heating and cooling of solids by forced convection and radiation is of considerable practical interest.

While the literature dealing with the limiting cases of forced convection and thermal radiation is extensive, the unsteady problem of heat conduction with combined convection and radiation heat transfer at the surface has received little attention. Transient heat conduction in a plate subjected to pure thermal radiation on one face and combined forced convection and radiation on the other has been studied by Burka [1]. The problem was reduced to the solution of two simultaneous, singular nonlinear Volterra integral equations which describe the fluxes at the two faces. The two equations were replaced by a system of nonlinear algebraic equations and were solved for one set of parameters over a limited time range. Burka [2] also studied transient heat conduction in a two dimensional rectangular region with convection prescribed on two faces and radiation on the remaining two. Vidin and Ivanov [3] have investigated the transient, symmetric heating of a plate by radiation and forced convection. The linear energy equation and nonlinear boundary condition were transformed into a nonlinear partial differential equation with a linear boundary condition. The resulting nonlinear partial differential equation was solved approximately by iteration. Some of the results of the analysis were compared with finite difference results. In regard to the limiting case of pure thermal radiation, an extensive literature review can be found in [4, 5].

Thus, it is the purpose of this paper to analyze the heating and cooling of a plate by combined forced convection and radiation and to obtain precise solutions so that the accuracy of various approximate schemes can be tested. Results are presented graphically. Asymptotic solutions are derived, and an approximation based on a linearized boundary condition is given for some parameters of physical interest. These approximations and the results available in the literature are compared with the "exact" solutions. The plate was considered because it was expected that the results for the temperature history would be between those for the semi-infinite solid and the infinitely long cylinder or sphere.

ANALYSIS

Statement of the problem

This investigation is concerned with the transient temperature distribution in a plate, initially at a uniform temperature, and suddenly subjected to thermal radiation and forced convection heat transfer at the surface. The following assumptions are made in the analysis:

1. The heat conduction is one dimensional.
2. The plate is isotropic, homogeneous, and opaque to thermal radiation.
3. The physical properties are independent of temperature.
4. The environment and fluid temperatures are not functions of time.
5. The heat-transfer coefficient is independent of surface temperature.
6. The plate does not contain any heat sources or sinks.
7. The fluid is transparent to thermal radiation.

The equation for the transient temperature distribution in the plate is

$$\frac{\partial T}{\partial t_1} = \alpha \frac{\partial^2 T}{\partial x_1^2}. \tag{1}$$

Assuming the initial temperature distribution in the plate is constant, the initial condition is as follows:

$$T(x_1, 0) = T_i. \tag{2a}$$

The heating or cooling is considered to be symmetrically about the center plane ($x_1 = 0$). This is arbitrary, and the assumption was introduced only to eliminate the independent parameters which are not essential to the understanding of the problem. Because of symmetry the temperature gradient at the center of the plate is zero:

$$\left.\frac{\partial T}{\partial x_1}\right|_{x_1=0} = 0. \tag{2b}$$

The final boundary condition is just a statement of an energy balance at the surface in contact with the fluid. The left-hand side of equation (2c) represents heat transfer by conduction, while the first term on the right-hand side accounts for radiation and the second for the convection of heat.

$$q(T_s) = -k\left.\frac{\partial T}{\partial x_1}\right|_{x_1=L} = \mathscr{F}_{se}\sigma(T_s^4 - T_e^4) + h(T_s - T_f). \tag{2c}$$

In writing the last boundary condition it has been assumed that the surface of the plate is gray and diffuse, and that the radiation incident on the surface is uniform over the surface.

To simplify the analysis, the basic equation (1) is written in terms of dimensionless variables

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \tag{3}$$

and the initial and boundary conditions take the form:

$$u(x, 0) = u_i \tag{4a}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0 \tag{4b}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=1} = -g(u_s). \tag{4c}$$

The dimensionless temperatures and the heat flux are defined in Table 1. The transient heating and cooling situations are treated separately to take into account the two limiting cases of $T_i = 0$ and $T_a = 0$.

Note that the more general case in which the fluid temperature differs from the environment temperature is included in the analysis. The surface flux can be written as

$$q = \mathscr{F}_{se}\sigma T_s^4 + hT_s - (\mathscr{F}_{se}\sigma T_e^4 + hT_f). \tag{5}$$

The group of parameters in the parenthesis determines the adiabatic surface temperature, T_a, defined by $q(T_a) = 0$. Thus, it is convenient to express the surface flux in terms of the adiabatic surface temperature

$$q(T_s) = \mathscr{F}_{se}\sigma(T_s^4 - T_a^4) + h(T_s - T_a), \tag{6}$$

where

$$\mathscr{F}_{se}\sigma T_a^4 + hT_a = \mathscr{F}_{se}\sigma T_e^4 + hT_f. \tag{7}$$

Formulation of integral equations

The partial differential equation (3) and boundary conditions (4) can be transformed to a singular nonlinear Volterra integral equation of the second kind by use of the Laplace transform with respect to time t and application of the convolution theorem. The details can be found elsewhere [4, 5] and therefore need not be repeated here. The resulting equation takes the form

$$u(x, t) = u_i - \int_0^t F(x, t - \tau)\, g[u_s(\tau)]\, d\tau \tag{8}$$

where

$$F(x, t - \tau) = 1 + 2\sum_{k=1}^{\infty} (-1)^k \cos(k\pi x) \times \exp[-k^2\pi^2(t - \tau)]. \tag{9}$$

Once the surface temperature of the solid has been determined, the temperature at any point and time can be calculated from equation (8) by simple quadrature. To solve for the surface temperature, x is set equal to unity. The kernel, $F(x, t - \tau)$, then reduces to the function

$$f(t - \tau) = 1 + 2\sum_{k=1}^{\infty} \exp[-k^2\pi^2(t - \tau)] \tag{10}$$

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and equation (8) for the surface temperature $[y(t) = u_s(t)]$ becomes

$$y(t) = u_i - \int_0^t f(t - \tau)\, g[y(\tau)]\, d\tau. \qquad (11)$$

Thus, the problem of transient heating or cooling of a plate by combined convection and thermal radiation has been reduced to solving a nonlinear Volterra integral equation for the surface temperature.

Table 1. Dimensionless variables

Process	$u(x, t)$	u_i	$u_s(t)$	$g(u_s)$
Cooling	$\frac{T(x, t)}{T_i}$	1	$\frac{T(1, t)}{T_i}$	$N_{rc}(u_s^4 - \theta_c^4) + N_{Bi}(u_s - \theta_c)$
Heating	$\frac{T(x, t)}{T_a}$	θ_h	$\frac{T(1, t)}{T_a}$	$N_{rh}(u_s^4 - 1) + N_{Bi}(u_s - 1)$

Method of solution

To illustrate the method of solution, the problem of cooling a plate, given in equation (11), will be considered. First, the time range over which the problem is to be solved is divided into, i, number of intervals. An initial guess is made for the first time interval, $t_0 = 0 \leqslant t \leqslant t_1$. This initial guess is improved by the method of successive approximations until the error between the last two approximations is less than a predetermined value. The $n + 1$ approximation for this interval is given below

$$y_{n+1}(t) = 1 - \int_0^t f(t - \tau)\, g[y_n(\tau)]\, d\tau. \qquad (12)$$

For the second interval $(t_1 \leqslant t \leqslant t_2)$, the integral is broken into two parts, as shown in the next equation

$$y_{n+1}(t) = 1 - \int_0^{t_1} f(t - \tau)\, g[y(\tau)]\, d\tau - \int_{t_1}^{t} f(t - \tau)\, g[y_n(\tau)]\, d\tau. \qquad (13)$$

Since $y(t)$ is known for $0 \leqslant t \leqslant t_1$, the first integral on the right side of equation (13) is known for all time t. Then, an approximation is made for the time interval, $t_1 \leqslant t \leqslant t_2$. Again, the method of successive approximations is used until the difference between the last two approximations is less than a predetermined error. For the m-th time interval the $n + 1$ approximation is

$$y_{n+1}(t) = 1 - \int_0^{t_{m-1}} f(t - \tau)\, g[y(\tau)]\, d\tau - \int_{t_{m-1}}^{t} f(t - \tau)\, g[y_n(\tau)]\, d\tau \quad (t_{m-1} \leqslant t \leqslant t_m). \qquad (14)$$

This process is continued down the time scale, until the surface temperature is determined for all desired time.

When the integrations are carried out numerically, the difficulty introduced by the singular kernel is removed by use of the Poisson summation formula [6]

$$f(t - \tau) = 1 + 2 \sum_{k=1}^{\infty} \exp[-k^2\pi^2(t - \tau)] = \left\{1 + 2 \sum_{k=1}^{\infty} \exp[-k^2/(t - \tau)]\right\} \times [\pi(t - \tau)]^{-\frac{1}{2}} \qquad (15)$$

and by application of the modified Gaussian integration formula [7]

$$\int_a^b f(z)(b - z)^{-\frac{1}{2}}\, dz = 2(b - a)^{\frac{1}{2}} \sum_{i=1}^{n} w_i^{(2n)} f(z_i) \qquad (16)$$

where $z_i = a + (b - a)(1 - x_i^2)$ and $w_i^{(2n)}$ are the Gaussian weights of order $2n$.

The initial approximation in the method of solution is of importance. In general, the closer the initial approximation to the exact solution, the faster the method of successive approximations converges to the exact solution. Thus, methods of obtaining these initial approximations are of interest.

The initial approximations were obtained by the intersection method [4, 5], which resulted in solving the following transcendental equation for $y(t)$

$$y(t) = 1 - \int_0^{t_{m-1}} f(t - \tau)\, g[y(\tau)]\, d\tau - g[y(t)] \int_{t_{m-1}}^{t} f(t - \tau)\, d\tau \quad (t_{m-1} \leqslant t \leqslant t_m). \qquad (17)$$

Asymptotic solutions

In this section, asymptotic solutions of equation (11) are considered for small and large times. These limiting cases are of practical interest because they yield simplified solutions and provide bounds to the exact solution.

For very small times, the method of successive substitutions is very useful. The initial approximation can be expressed as follows:

$$y(t) = u_i - g(u_i) \int_0^t f(\tau)\,d\tau. \tag{18}$$

The result is equivalent to assuming that the radiative and convective flux at the surface is constant and equal to $g(u_i)$.

Using the Poisson summation formula the integral equation (11) describing the surface temperature can be written as

$$y(t) = u_i - \int_0^t \frac{g[y(\tau)]}{\sqrt{[\pi(t-\tau)]}}\,d\tau - 2\sum_{k=1}^{\infty}\int_0^t \frac{g[y(\tau)]\exp[-k^2/(t-\tau)]\,d\tau}{\sqrt{[\pi(t-\tau)]}}. \tag{19}$$

Since $\exp(-k^2/t) \geqslant \exp[-k^2/(t-\tau)]$, the last term in equation (19) is bounded as follows

$$2\sum_{k=1}^{\infty}\int_0^t \frac{g[y(\tau)]\exp[-k^2/(t-\tau)]}{\sqrt{[\pi(t-\tau)]}}\,d\tau \leqslant g(u_i)\frac{4\sqrt{t}}{\sqrt{\pi}}\sum_{k=1}^{\infty}\exp(-k^2/t) = \epsilon_{\max}(t). \tag{20}$$

As can be seen from Table 2, the surface temperature for small time can be approximated by

$$y(t) = u_i - \int_0^t \frac{g[y(\tau)]}{\sqrt{[\pi(t-\tau)]}}\,d\tau. \tag{21}$$

This nonlinear integral equation represents the surface temperature of a semi-infinite solid subject to combined thermal radiation and forced convection [5]. For small times ($t \ll 1$) the series

$$y(t) = \sum_{j=0}^{\infty} a_j t^{j/2}$$

provides good results. The coefficients, a_j, of the series are determined by substituting the series into both sides of equation (21) and comparing the coefficients of equal powers of t. For example, in the case of cooling the first two coefficients are $a_0 = 1$ and

$$a_1 = 2[N_{rc}(1-\theta_c^4) + N_{Bi}(1-\theta_c)]/\sqrt{\pi}.$$

Another useful small time approximation is based on the intersection method [4, 5] and results in the transcendental equation

$$y(t) = u_i - g[y(t)]\int_0^t f(\tau)\,d\tau. \tag{22}$$

This approximation is equivalent to truncating the Taylor series expansion of $g[y(\tau)]$ after the first term

$$g[y(\tau)] = g[y(t)] + (\tau - t)g'[y(t)] + \frac{(\tau-t)^2}{2}g''[y(t)] + \dots. \tag{23}$$

Increased accuracy can be obtained by including more terms of the expansion in the analysis, but the resulting differential equation is singular. For example, including the second term results in the following differential equation

$$dg[y(t)]/dt = \{y(t) - u_i + g[y(t)] \times \int_0^t f(\tau)\,d\tau\}/\int_0^t \tau f(\tau)\,d\tau \tag{24}$$

with initial condition $y(0) = u_i$.

The large time solution for equation (11) corresponds to the case of infinite thermal conductivity. For large time equation (11) reduces to the following form

$$y(t) = u_i - \int_0^t g[y(\tau)]\,d\tau \tag{25}$$

because

$$\lim_{t\to\infty}\int_0^t g[y(\tau)]\{\sum_{k=1}^{\infty}\exp[-k^2\pi^2(t-\tau)]\}\,d\tau = 0. \tag{26}$$

Table 2.

t	$\epsilon_{max}(t)/g(u_i)$
0·025	$0{\cdot}151 \times 10^{-17}$
0·050	$0{\cdot}104 \times 10^{-8}$
0·075	$0{\cdot}100 \times 10^{-5}$
0·100	$0{\cdot}324 \times 10^{-4}$
0·150	$0{\cdot}111 \times 10^{-2}$
0·200	$0{\cdot}680 \times 10^{-2}$
0·300	$0{\cdot}441 \times 10^{-1}$

Differentiating the integral equation with respect to t, yields the following differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -g(y) \tag{27}$$

with initial condition $y(0) = u_i$. The above equation can be solved analytically, but usually this procedure is more burdensome than a numerical solution. For the case of cooling to a zero adiabatic temperature, the solution to the differential equation is

$$y(t) = \left[\frac{N_{Bi}\exp(-3N_{Bi}t)}{N_{Bi} + N_{rc} - N_{rc}\exp(-3N_{Bi}t)}\right]^{\frac{1}{3}}. \tag{28}$$

DISCUSSION OF RESULTS

Since the boundary condition is nonlinear, it is impossible to non-dimensionalize the equations and still avoid the necessity for considering the cooling and heating cases separately. When the problem is non-dimensionalized in a manner described earlier, the unsteady heating or cooling of a plate is essentially a three parameter problem. Moreover, the numerical solution of the problem for the set of parameters (N_{Bi}, N_{rc} and θ_c) for cooling and of the parameters (N_{Bi}, N_{rh} and θ_h) for heating is an undertaking of considerable magnitude. Therefore, only a few representative calculations were performed. The cases chosen for study correspond to those where convection and radiation are of the same order of magnitude and where the heat transfer by one mode is fixed and the other varies over a considerable range. Once the surface temperature is known for all time the temperature distribution and the heat flux can be readily calculated from equation (8). The latter, however, is not very informative and is therefore not included for the sake of brevity. The surface temperatures were calculated to an accuracy of at least four significant figures.

Figures 1 and 2 show the effects of the dimensionless initial temperatures θ_c and θ_h. The results given in these two figures correspond to the case where heat transfer by convection and radiation, at least initially, are of about the same order of magnitude. We note that in Fig. 1 the curves for $\theta_c > 1$ correspond to heating and those in Fig. 2 for $\theta_h > 1$ correspond to cooling. For example, in the case of cooling, this is due to the fact that the adiabatic surface temperature T_a is smaller than the initial temperature T_i. It can be seen from the figures that at large times the trends of the surface temperatures are quite different. In Fig. 2, all the curves collapse for values of dimensionless surface temperature close to unity. In both figures, as the dimensionless initial temperature is increased the time needed to obtain a certain surface temperature is decreased.

The effect of the Biot number is illustrated in Figs. 3 and 4, while the effect of the radiation number is presented in Figs. 5 and 6. The results are for the case when θ_c or θ_h is equal to zero. While physically the adiabatic or initial temperature cannot be equal to absolute zero, in many situations the temperature ratio θ_c or θ_h can be very small. The choice of these parameters is of course arbitrary; however, as can be seen from Figs. 1 and 2 a small variation from zero has little effect on the results. Also, the nonlinearity is most pronounced when $\theta_c = 0$ or $\theta_h = 0$. As would be expected, the pure convection and pure radiation cases act as lower bounds for the combined radiation and convection solutions. An increase in one of the two parameters results in a decrease in the time required to reach a given dimensionless temperature. For cooling, a change in the Biot number produces greater variations in the surface temperature than an equal change in

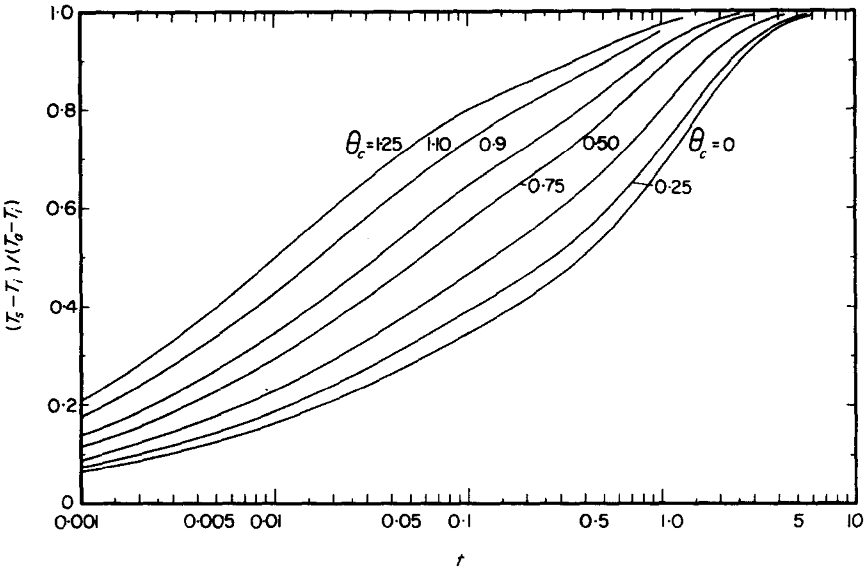

FIG. 1. Effect of θ_c on the surface temperature of a plate subjected to convective and radiative heat transfer, $N_{rc} = N_{Bi} = 1$.

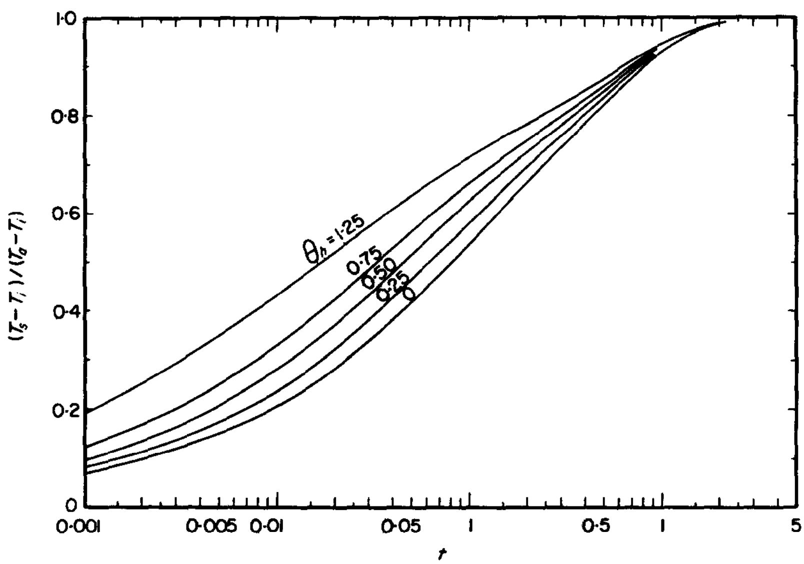

FIG. 2. Effect of θ_h on the surface temperature of a plate subjected to convective and radiative heat transfer, $N_{rh} = N_{Bi} = 1$.

the radiation number, while the opposite is true for heating.

The convective mode of heat transfer appears to be dominant as the dimensionless temperature approaches unity for a plate cooling to a zero environment temperature. This result is due to the fact that y^4 is approaching zero at a much faster rate than y. For example, when y is equal to ϵ (a small number), the dimensionless radiative flux would be $\epsilon^4 N_{rc}$ while the

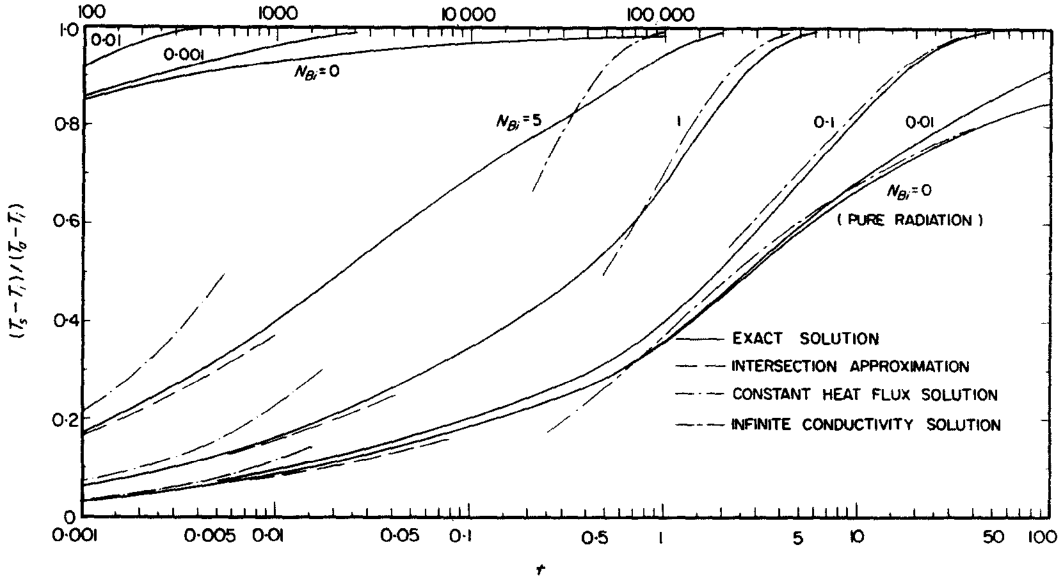

FIG. 3. Effect of the Biot number on the surface temperature of a plate cooled by convection and radiation, $N_{rc} = 1$, $\theta_c = 0$.

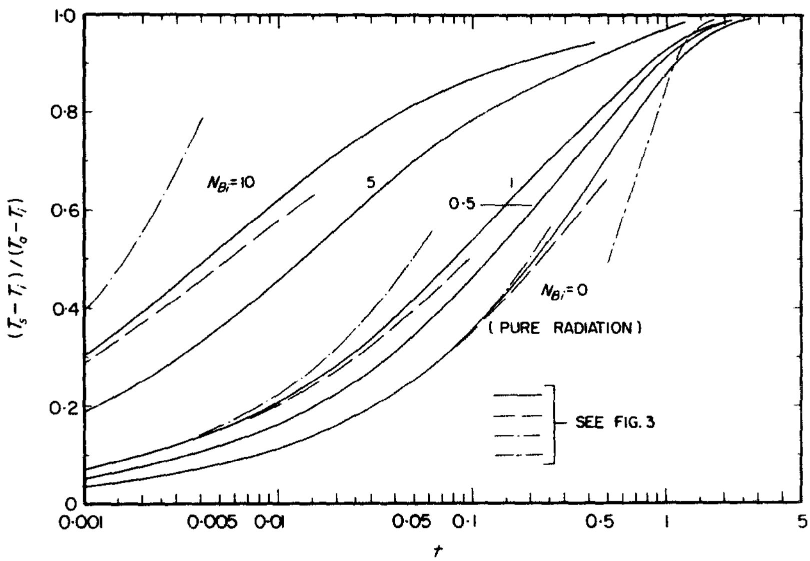

FIG. 4. Effect of the Biot number on the surface temperature of a plate heated by convection and radiation, $N_{rh} = 1$, $\theta_h = 0$.

convective flux would be ϵN_{Bi}. Thus, their ratio would be $\epsilon^3(N_{rc}/N_{Bi})$, and only when the ratio N_{rc}/N_{Bi} is large, would the convection flux fail to be dominant. For heating, the convective and radiative fluxes are more nearly equal. When $y = 1 - \epsilon$, the dimensionless radiative flux and the convective flux are $4\epsilon N_{rh}$ and ϵN_{Bi}, respectively. Thus, the radiation would tend to be somewhat more dominant than the convection.

The assumption that if N_{Bi}/N_{rc} is small, the pure radiation solution will provide a good

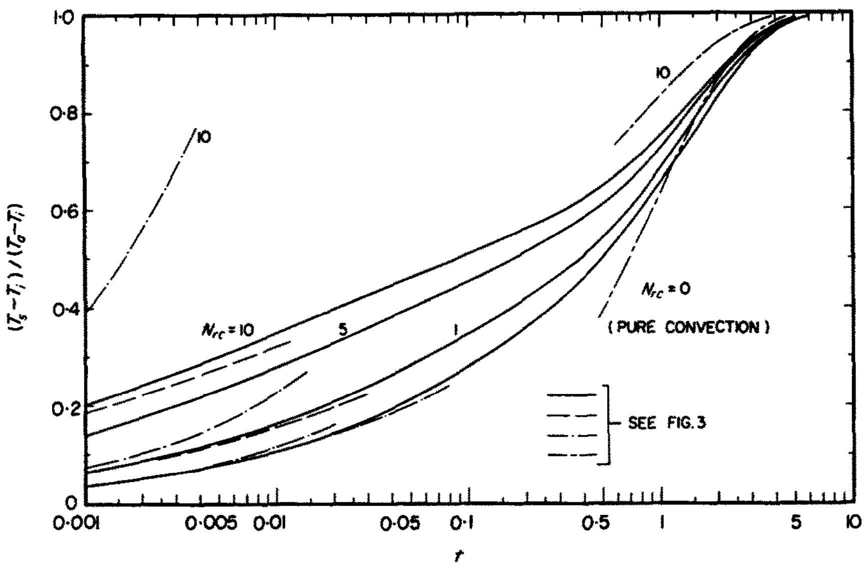

FIG. 5. Effect of the radiation number on the surface temperature of a plate cooled by convection and radiation, $N_{Bi} = 1$, $\theta_c = 0$.

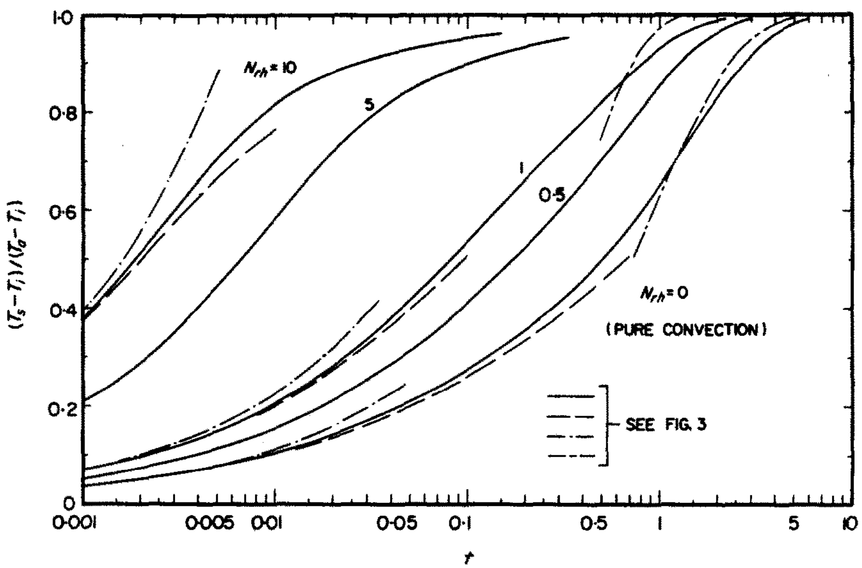

FIG. 6. Effect of the radiation number on the surface temperature of a plate heated by convection and radiation, $N_{Bi} = 1$, $\theta_h = 0$.

approximation may be inaccurate for large Fourier number. This is illustrated by considering the plate cooling to a zero environment temperature with $N_{Bi} = 0{\cdot}01$ and $N_{rc} = 1$. The dimensionless time required to cool the plate to 1 per cent of its initial temperature is 310, while cooling by pure radiation would require a Fourier number of 340000. Even if the Biot number was 0·001 it would only take a Fourier number of 2300. For large times and a small Biot number let $N_{Bi}/N_{rh} = N_{Bi}/N_{rc} = \delta$ and set $y = \epsilon$ for cooling and $y = 1 - \epsilon$ for heating, where ϵ and δ are numbers much smaller than unity. The ratio of convective flux to radiative flux is

$$\frac{\epsilon N_{Bi}}{\epsilon^4 N_{rc}} = \frac{\delta}{\epsilon^3} \quad \text{for cooling}$$

and

$$\frac{\epsilon N_{Bi}}{4\epsilon N_{rh}} = \frac{\delta}{4} \quad \text{for heating.}$$

Thus, for heating the surface temperature is not expected to behave in the same manner as for cooling. The results in Fig. 6 confirm this expectation.

The results indicate that the infinite thermal conductivity solution gives useful results for $t \geqslant 10$. The constant heat flux approximation and intersection method provide good results for small time. The accuracy of these small time solutions decreases as the Biot number or radiation number is increased. Of the two, the intersection method is the more accurate. It should be noted that the solution based on the constant heat flux approximation is unbounded, while the intersection method yields a solution which is bounded.

A common linearization technique employed in problems of this type is to replace the radiation term by a convective flux. Accordingly the surface flux for heating is written in the form

$$g \approx \left[N_{rh}\frac{(u_{AV}^4 - 1)}{u_{AV} - 1} + N_{Bi}\right](u_s - 1)$$

where $u_{AV} = (1 + \theta_h)/2$ and for cooling

$$g \approx \left[N_{rc}\frac{(u_{AV}^4 - \theta_c^4)}{u_{AV} - \theta_c} + N_{Bi}\right](u_s - \theta_c)$$

where $u_{AV} = (1 + \theta_c)/2$. The definition of u_{AV} is somewhat arbitrary. For specific cases, a more meaningful average might be selected. Obviously, the procedure is not satisfactory for all values of the parameters. In Fig. 7 a few representative results are presented for cooling. As expected, for pure radiation large deviations are noted between the two solutions. When the radiation and Biot numbers are equal to unity, the accuracy of the linearized solution is improved. As the environment temperature approaches unity, $\theta_c \to 1$, the deviation between the linearized and exact solution decreases. Similar trends are illustrated in Fig. 8 for heating. For both heating and cooling the solutions are initially inaccurate due to the fact that at $t = 0$

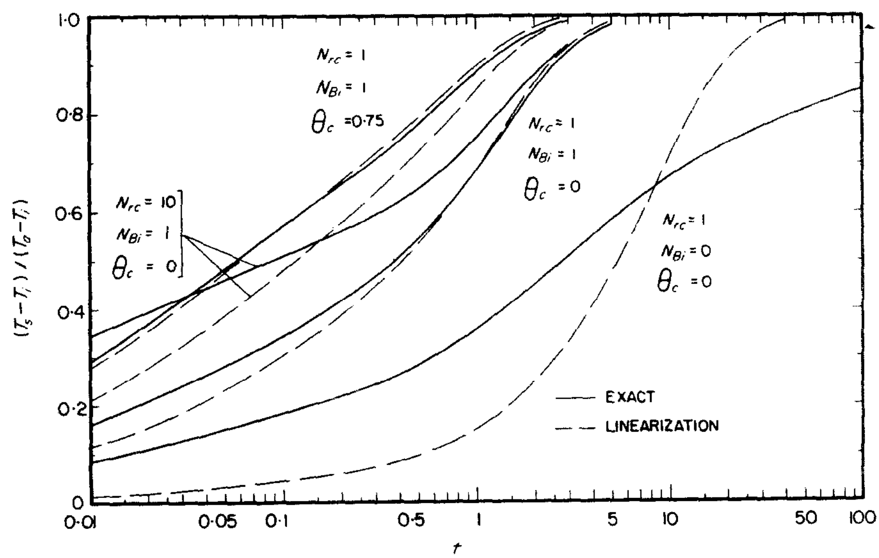

FIG. 7. Comparison of results for cooling.

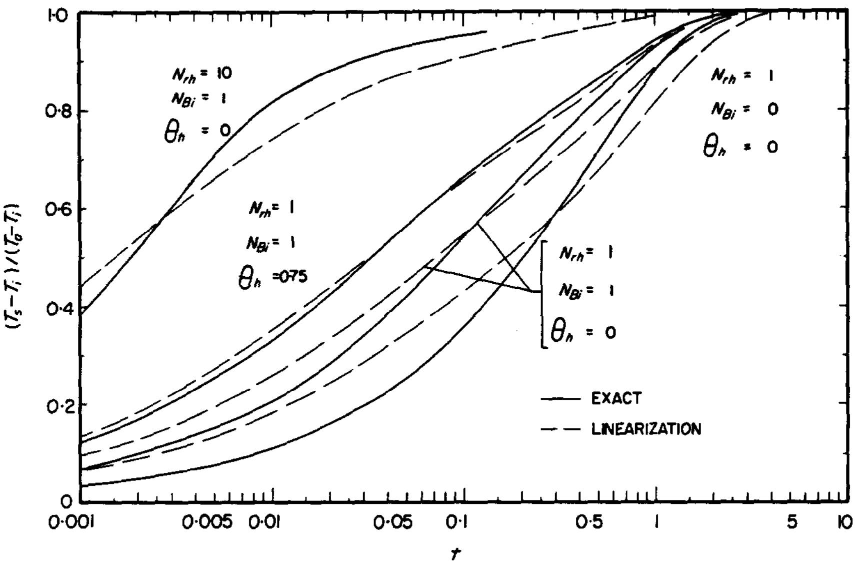

FIG. 8. Comparison of results for heating.

the linearized heat flux is not equal to the actual flux. Thus, care must be exercised in applying the linearization technique.

It is also of interest to examine the accuracy of the approximate analytic solution proposed by Vidin and Ivanov [3]. This method and one of numerical integration of equations (3) and (4) are compared with the technique proposed in this paper. The comparison is given in Table 3.

Table 3. Comparison of approximations with exact solution for $N_{rh} = 0{\cdot}4$, $N_{Bi} = 0{\cdot}5$ and $\theta_h = 0{\cdot}2$

t	Analytic approximation [3]	Finite difference [3]	Linearization	Present method
0	0·2000	0·2000	0·2000	0·2000
0·25	0·5372	0·5577	0·5759	0·5563
0·50	0·6612	0·6707	0·6679	0·6670
0·75	0·7554	0·7541	0·7365	0·7488
1·00	0·8288	0·8176	0·7907	0·8115
1·25	0·8831	0·8652	0·8338	0·8592
1·50	0·9219	0·9012	0·8680	0·8951
1·75	0·9486	0·9269	0·8951	0·9219
2·00	0·9664	0·9462	0·9167	0·9419

Vidin and Ivanov pointed out that as N_{rh} and N_{Bi} are decreased, the accuracy of their method increases.

CONCLUDING REMARKS

A practical method for determining precise solutions for the temperature distribution in a plate subjected to combined convective and radiative heat flux at the surface has been developed. The method is "exact" in the sense that any degree of accuracy may be obtained and the error estimated. The method is not intended for heat-transfer calculations where only ordinary engineering accuracy is required.

It should be pointed out that the present method of solution can also be used for other one-dimensional geometries [4, 5]. By way of generalization, we should like to remark that the analysis could be extended to other transient heat-conduction problems such as when the initial temperature is nonuniform, heat is generated in the solid, heat-transfer coefficient is a function of surface temperature and/or time, and the ambient fluid and environment temperatures vary with time. The method of analysis could also be adopted for the solution of steady slug flow in channels which are heated or cooled by combined convection and radiation if the temperature gradient across the wall of channel is assumed to be negligible.

From the results presented in the paper the following observations can be made:

1. The neglect of one mode of heat transfer in favor of the dominant one, even though this may appear to be warranted from the examination of the dimensionless parameters, can lead to appreciable errors.
2. The linearization of the surface heat flux yields reasonable results only when convection predominates radiation and when the surface temperature is close to the environment temperature. The procedure should be used with caution.

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Résumé—On a étudié la conduction de la chaleur un régime transitoire dans une plaque soumise à un chauffage et/ou à un refroidissement par de la convection combinée avec le rayonnement. La plaque est supposée être homogène, isotrope et opaque au rayonnement thermique. La distribution de température initiale et l'ambiance ainsi que les températures du fluide sont supposées constantes. Dans l'analyse, l'équation de conduction de la chaleur en régime transitoire et les conditions aux limites sont transformées en une équations intégrale non-linéaire de Volterrade seconde espèce pour la température de la surface.

Cette équation est résolue numériquement sur un calculateur numérique par la méthode des approximations successives. On a obtenu des résultats précis au moins jusqu'au quatrième chiffre significatif dans une gamme de paramètres d'intérêt physique. Ils sont présentés graphiquement et comparés avec des solutions approchées et des résultats disponibles dans la littérature. On donne également des solutions asymptotiques pour des temps petits et grands et l'on discute leur région de validité.

Zusammenfassung—Es wurde die durch Heizen und/oder Kühlen infolge gleichzeitiger Konvektion und Strahlung hervorgerufene instationäre Wärmeleitung in einer Platte untersucht. Es wird angenommen, dass die Platte für die Wärmestrahlung homogen, isotrop und undurchlässig ist. Die Anfangstemperaturverteilung, die Temperatur der Umgebung und die Flüssigkeitstemperaturen werden als konstant angenommen. Bei der Berechnung werden die Gleichung für die instationäre Wärmeleitung und die Randbedingungen in eine nichtlineare Volterra-Integralgleichung zweiter Art für die Oberflächentemperatur transformiert.

Diese Gleichung wird mit Hilfe eines digitalen Elektronenrechners mit der Methode aufeinanderfolgender Näherungen gelöst.

Аннотация—Исследуется нестационарная теплопроводность нагреваемой или охлаждаемой пластины при совместной конвекции и излучении. Пластина считается однородной, изотропной и проницаемой для теплового излучения. Начальное распределение температур, температура окружающей среды, а также температура жидкости считаются постоянными. При анализе уравнения нестационарной теплопроводности и граничные условия преобразуются в нелинейное интегральное уравнение Вольтерра второго рода для поверхностной температуры. Это уравнение решается численно на цифровой вычислительной машине методом последовательных приближений. Получены результаты с точностью до четвертой значащей цифры для параметров, представляющих интерес с физической точки зрения. Они представлены графически в сравнении с приближенными решениями и известными из литературы результатами. Приводятся асимптотические решения для больших и малых значений времени и обсуждается область их применения.